AMSAA

TECHNICAL REPORT NO. [illegible]

METHODOLOGY FOR ACCEPTANCE OF [illegible]
TARGET DISPERSION CHARACTERISTICS [illegible]
ARMOR PIERCING DISCARDING SABOT [illegible]
ROUNDS

FRANK [illegible]

SEPTEMBER 1977

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TR-194 ✓	2. GOVT ACCESSION NO. (14) AMSAA-TR-194	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Methodology for Acceptance Criteria for Target Dispersion Characteristics of the Armor Piercing Discarding Sabot (APDS) Rounds,		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Frank C. Hopkins		8. CONTRACT OR GRANT NUMBER(s) (12) 42p.
9. PERFORMING ORGANIZATION NAME AND ADDRESS U S Army Materiel Systems Analysis Activity ✓ Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project No. 1R 765706M541
11. CONTROLLING OFFICE NAME AND ADDRESS U S Army Materiel Development & Readiness Command 5001 Eisenhower Avenue, Alexandria, VA 22333		12. REPORT DATE (11) Sep[redacted]77
		13. NUMBER OF PAGES 49
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (9) Technical rept.,		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; unlimited distribution.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acceptance Criteria	105mm: APDS, M392
Operating Characteristics	Circular Probable Error
Target Dispersion	Gamma Distribution
Accuracy	Chi-square Distribution
Probable Error	Dispersion Characteristics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report develops methodology for acceptance criteria for target dispersion characteristics of Armor Piercing Discarding Sabot (APDS) rounds. The distribution of observed circular probable errors of the reference lot of Cartridge, 105mm: APDS-T M392 is obtained and is used to derive the operating characteristics (OC) curves of several acceptance plans. Procedures for developing acceptance plans and their associated O-C curves are given, and several plans are presented.

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ACRONYMS AND SYMBOLS

APDS	Armor Piercing Discarding Sabot
mm	Millimeter
OC	Operating Characteristics
PE	Probable Error
CPE_c	Characteristic Circular Probable Error
$\hat{CPE}_c$	Estimate of Characteristic Circular Probable Error
CPE_g	Level of Circular Probable Error which represents good lot quality
CPE_I	Inherent Circular Probable Error
CPE_L	Circular Probable Error which characterizes a lot
CPE_o	Observed Circular Probable Error
K	A Constant
λ	Ratio of CPE_o to CPE_c
n	Sample Size
N	Number of occasions
$\hat{PE}$	Estimate of Probable Error
σ^2	Population Variance
σ_x^2, σ_y^2, σ_R^2, etc.	Population Variance of a set of x, y and radial observations, etc.
$\hat{\sigma}^2$	Estimate of σ^2
$\hat{\sigma}_x^2$, $\hat{\sigma}_y^2$, $\hat{\sigma}_R^2$, etc.	Estimates of σ_x^2, σ_y^2, σ_R^2, etc.
S^2	Calculated variance of a sample
S_x^2, S_y^2, S_R^2, etc.	Calculated variance of a set of x, y and radial observations, etc.

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METHODOLOGY FOR ACCEPTANCE CRITERIA FOR TARGET DISPERSION CHARACTERISTICS OF THE ARMOR PIERCING DISCARDING SABOT (APDS) ROUNDS

1. INTRODUCTION

1.1 General

The estimation of target dispersion characteristics of Armor Piercing Discarding Sabot (APDS) ammunition in acceptance testing is accompanied by a number of difficulties. Test costs are high; therefore sample sizes are limited. Since dispersion patterns are subjected to relatively large variations, small sample sizes produce undesirable levels of inaccuracy in estimating dispersion characteristics. Control of test conditions throughout the test is limited to control of only a few factors such as tube aim point, cant, stability of the firing platform and tube condition, at the start of the test. Even with maximum possible control of such factors, they still exert a degree of error in round-to-round target impact points. In addition, there are several uncontrollable factors, such as wind and weather conditions, tube wear from round to round, droop, jump and other unknowns, which make it impossible to obtain uniform conditions throughout the test. These problems have accompanied every acceptance test conducted on APDS ammunition and have been exacerbated by two factors:

- o The lack of established test procedures designed to minimize the effects of uncontrollable test condition variations and

- o The lack of established acceptance criteria and estimation procedures designed to minimize consumer and producer risks.

The result has been that a large number of lots of APDS ammunition with very poor target dispersion characteristics have been accepted for use.

1.2 Purpose

This report develops methodology which can be used to derive acceptance plans for target dispersion characteristics of APDS rounds. In developing the methodology, the effect of test condition variations upon target dispersion patterns and the lack of established acceptance criteria and estimation procedures are addressed. Examples of inadequate firing procedures in accuracy tests of APDS rounds are presented, and corrective measures are proposed. Examples of acceptance criteria and estimation procedures which minimize consumer's and producer's risks are developed. Several acceptance plans, derived from the proposed methodology are presented.

2. TEST PROCEDURES

2.1 Background

The objective of an accuracy acceptance test of APDS rounds is to assess the dispersion pattern which characterizes a lot of ammunition and accept or reject the lot. The acceptance test requires firing a group of rounds at a vertical target some distance from the gun. The coordinates of the impact points of each round in the group are obtained, and estimates of the dispersion about the center of impact are determined.

When firing a group of rounds to assess dispersion characteristics, it is desirable to have identical test conditions for each round. In this manner, the differences in impact points of each round are the result only of inherent differences between the rounds. Inherent differences between rounds in a lot of APDS ammunition are due to chance variation within a stable pattern caused by manufacturing procedures and physical characteristics of the round and propelling charge. These inherent differences lead to different flight characteristics, and cause rounds to impact at different points on the target. If identical test conditions are obtained, the dispersion characteristics of the group of rounds fired reflect the degree of round-to-round uniformity in the manufacturing process and provide an estimate of quality control.

Unfortunately, test conditions from round to round are not identical. Gun elevation and deflection vary regardless of efforts to maintain a constant aim point. Weather conditions and other factors which effect accuracy also vary from round to round. Consequently, the dispersion pattern of a group of rounds on a target is not representative of the inherent round-to-round differences. The dispersion pattern consequently represents the combination of the inherent differences in rounds and the variability in test conditions from round to round.

2.2 Firing Procedures

The method of firing employed in an acceptance test of a lot of APDS ammunition must be conducted in a way that minimizes the effect of round-to-round variability in test conditions. In past acceptance tests, gun elevation and deflection settings have been controlled to a great extent, and severe weather conditions have been avoided. However, the method of firing in acceptance tests has not been one which minimized round-to-round variability in test conditions. The length of time required to fire a group of rounds has been as great as four hours. Test conditions such as tube droop, cant, ambient environmental conditions and other unknowns vary more over a long time interval than they do in a short one. Hence, as shown by an analysis of past acceptance tests of 105mm, APDS ammunition, a group of rounds fired over a long time period will tend to exhibit higher probable errors than would be observed over a short time period.

Figure 1 shows the accuracy results of an acceptance test of 105mm, APDS rounds conducted at Jefferson Proving Ground. Horizontal and vertical probable errors are presented as a function of time between rounds. The wind ranged from 2-10 knots and varied in direction from 140° to 180° during the course of the test. The probable errors for the entire 25 round group were calculated at 0.47 mils in the horizontal direction and at 0.29 mils in the vertical direction. Probable errors as a function of time between rounds were obtained by analyzing all combinations of two round groups in the test. The probable errors for each two round group were calculated and correlated with time between rounds fired. Although the trend in Figure 1 is linear, other shapes may be expected from the testing of other lots.

Figure 1 clearly illustrates that during an accuracy test the dispersion of impacts is greatly affected by the test conditions, which, in turn, vary with time.

The firing procedures employed in acceptance tests have not been designed to minimize the time over which a group of rounds is to be fired. In the past, as many as three different lots were often tested simultaneously, with rounds from each lot fired alternately, with reference rounds. The effect quadrupled the amount of time required to fire each test group of each lot. Consequently, the effect on dispersion due to variations in round-to-round test conditions, has been greater than that which could have been obtained if the time for firing each group were reduced.

In conducting an acceptance test, the individual groups of rounds from a test lot should be fired sequentially with no alternate firing of reference rounds or rounds from other test lots between rounds within a group. The time for firing each group of rounds can thus be minimized to the greatest extent possible. If reference rounds are to be fired, each group of reference rounds should be fired either before or after each group of test rounds. For example, if two ten round samples from a single lot are to be tested with fifteen reference rounds, the order of firing could be:

Five Reference Rounds

Ten Sample Rounds

Five Reference Rounds

Ten Sample Rounds

Five Reference Rounds

Estimates of probable errors for each group of rounds would then be calculated and pooled accordingly.

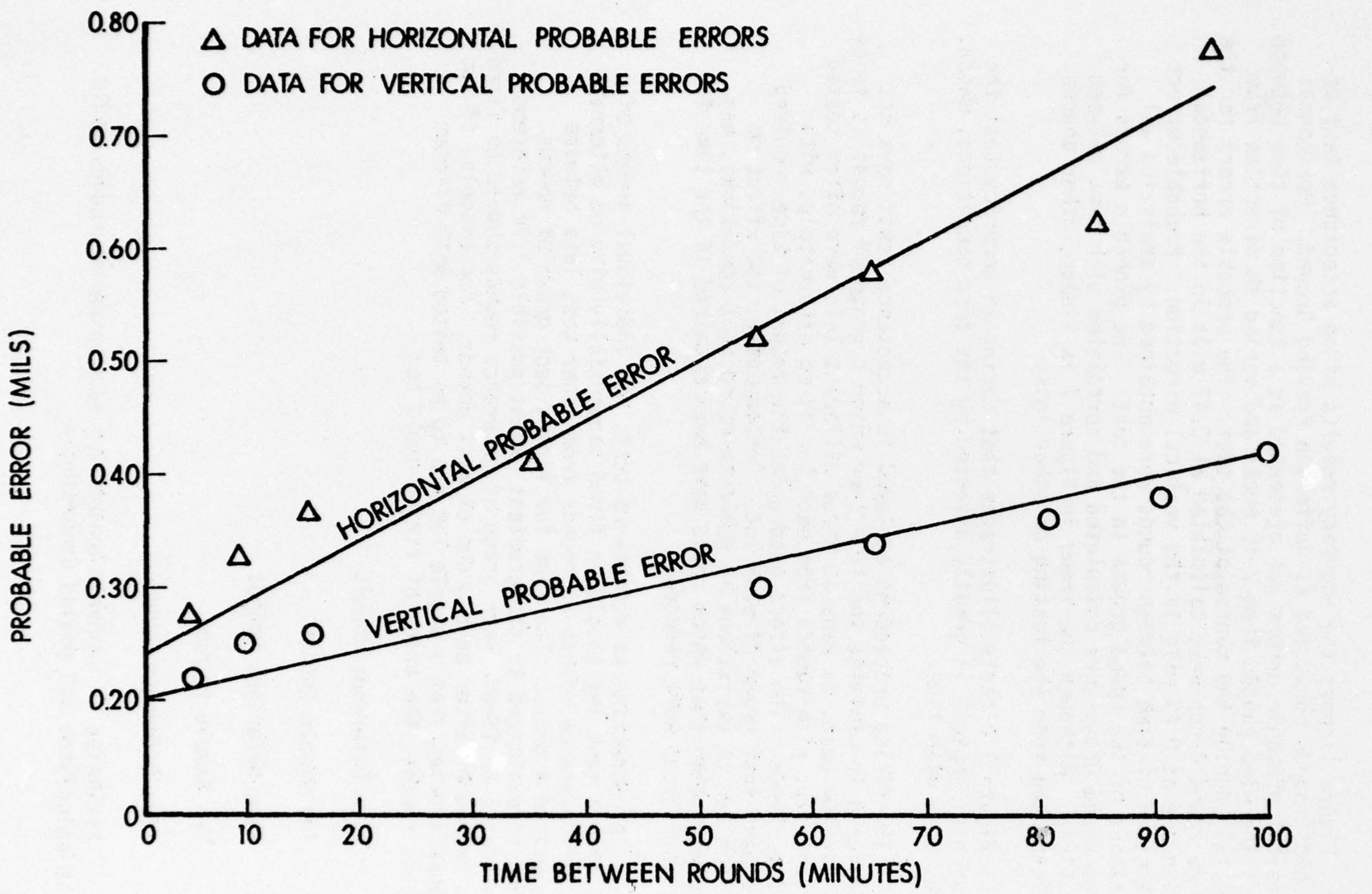

Figure 1. Probable Errors Observed During an Acceptance Test of a Single Lot of 105MM, M392, APDS as a Function of Time Between Rounds.

3. CALCULATION PROCEDURES

When a group of rounds is fired over a short time interval, the effect of nonuniform test conditions from round to round will still persist. Movement of the mean center of impact from round to round may occur, and, if so, the effect of this trend on calculated probable errors may be eliminated by the method of successive differences (Reference 1). For example, suppose the following impact coordinates, measured in inches, are obtained for a group of ten rounds fired at a vertical target 1000 meters from the gun:

Round Number	Horizontal Coordinate (x)	Vertical Coordinate (y)
1	65	110
2	70	120
3	60	115
4	75	100
5	70	105
6	85	95
7	80	90
8	95	95
9	90	85
10	100	75

Calculating probable errors in the usual manner for the entire ten-round group (References 2 and 3) yields,

Horizontal probable error = 0.23 mils

Vertical probable error = 0.24 mils

Circular probable error = 0.41 mils.

Calculating probable errors using the method of successive differences yields,

Horizontal probable error = 0.13 mils

Vertical probable error = 0.11 mils

Circular probable error = 0.21 mils.

The method of successive differences results in approximately a 50 percent reduction in probable error estimates in this particular example. The probable errors calculated by the standard method include the effects of test condition variability over the entire ten-round group, while the probable errors calculated by the method of successive

differences include only the test condition variability between successive rounds. Whenever the variation in test conditions is nonrandom, the result is a nonrandom dispersion pattern of shots about the center of impact. A trend of this type indicates that test condition variability over the entire group is greater than the variability between successive rounds. When this type of trend occurs, the use of the method of successive differences provides dispersion estimates which include only the effects of round-to-round test condition variability. In this manner, the effect of variability over the entire group is eliminated.

In determining the dispersion characteristics of a lot from a group of rounds, it is therefore desirable to limit the effect of test condition variability on the estimates of dispersion to that variability which occurs only between rounds.

It can be shown that the method of successive differences provides an unbiased estimate of the square of the probable error (PE). From statistical theory, an unbiased estimate of the variance ($\hat{\sigma}_x^2$) in the x direction of a two-round sample is obtained by

$$\hat{\sigma}_x^2 = \frac{1}{2}(x_1 - x_2)^2, \text{ where}$$

x_1 and x_2 are the coordinates of impact on the x axis. If three rounds are fired and have coordinates of impact x_1, x_2 and x_3 on the x axis,

$$S_{1x}^2 = \frac{1}{2}(x_1 - x_2)^2 \text{ and } S_{2x}^2 = \frac{1}{2}(x_2 - x_3)^2$$

provide two unbiased estimates of the variance, σ_x^2. The sample variance calculated by the method of successive differences is $S_x^2 = 1/2\ S_{1x}^2 + 1/2\ S_{2x}^2$, and it is an unbiased estimate of σ_x^2. Since

$$\begin{aligned} E(S_x^2) &= E(1/2\ S_{1x}^2 + 1/2\ S_{2x}^2) \\ &= E(1/2\ S_{1x}^2) + E(1/2\ S_{2x}^2) \\ &= 1/2\ E(S_{1x}^2) + 1/2\ E(S_{2x}^2) \\ &= \sigma_x^2/2 + \sigma_x^2/2 \\ &= \sigma_x^2. \end{aligned}$$

Similarly, for a sample of size n

$$S_{ix}^2 = 1/2\,(x_i - x_{i+1})^2, \; i = 1, 2, \ldots, n-1.$$

Then $S_x^2 = \frac{1}{n-1}\sum_{i=1}^{n-1} S_{ix}^2$, the sample variance calculated by the method of successive differences, provides an unbiased estimate of σ_x^2, because

$$E(S_x^2) = E\left(\frac{1}{n-1}\sum_{i=1}^{n-1} S_{ix}^2\right)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n-1} E(S_{ix}^2)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n-1} \sigma_x^2$$

$$= \sigma_x^2.$$

Since $(PE_x)^2 = K\sigma_x^2$,

where PE_x = probable error in the x direction and K is the appropriate constant,

$$E(\hat{PE}_x^2) = E(KS_x^2) = K\sigma_x^2 = PE_x^2,$$

where $\hat{PE}_x^2 = KS_x^2$ and $E(S_x^2) = \sigma_x^2$.

Hence, the method of successive differences gives an unbiased estimate of the square of the probable error between rounds in the x direction.

A similar proof shows that the method of successive differences also provides an unbiased estimate of the probable error squared between rounds in the y direction and also in the radial direction (circular probable error).

Consequently, the use of the method of successive differences provides unbiased estimates of the probable error squared and eliminates the effect of nonrandom test condition variation upon the dispersion results of an entire group of rounds.

Whenever the variation in test conditions is nonrandom and, thus, results in a dispersion pattern which is not random about the center of impact, the method of successive differences should be employed.

4. ACCEPTANCE CRITERIA

4.1 Applicable Parameters

In the development of acceptance criteria for dispersion characteristics of APDS rounds, parameters appropriate to an accept/reject decision must be selected. Horizontal and vertical probable errors could be the basis for a decision with independent criteria for each. This has the disadvantage, however, of not utilizing all available information. For example, the observed horizontal and vertical probable errors in a test could be 0.10 mils and 0.35 mils, respectively. The pooled average of these is 0.26 mils. If the reject criterion is to reject lots when either horizontal or vertical probable error is greater than 0.30 mils, this lot would be rejected. The problem with this type of accept/reject criteria is that it ignores good dispersion characteristics in one direction when dispersion in the other direction is poor.

Results of acceptance tests and life cycle evaluations of 105mm, M392, APDS rounds have shown that target dispersion patterns are approximately circular. In Reference 4, for example, horizontal and vertical probable errors were 0.19 mils and 0.21 mils, respectively for 803 rounds fired from a mid-life tube. Hence, the use of circular probable error, which effectively combines all dispersion information in both the horizontal and vertical directions, is appropriate. Use of this parameter simplifies the accept/reject criteria. Another advantage of using the circular probable error in estimating target dispersion rather than the present technique of computing independent horizontal and vertical probable errors, is that the sample size requirement is reduced significantly for the specified risk.

4.2 Distribution Of Circular Probable Errors

A lot has an inherent circular probable error, CPE_I, which describes its expected performance when random samples from the lot are fired under identical test conditions. If CPE_0 is the circular probable error observed for a random sample fired under identical test conditions, then

$$E(CPE_0) = CPE_I$$

Since circular probable error is a multiple of the radial standard deviation it follows that,

$$\frac{CPE_0^2}{CPE_I^2} = \frac{S_R^2}{\sigma_R^2},$$

where S_R is the observed radial standard deviation, and σ_R is the expected radial standard deviation for a random sample fired under identical conditions.

The distribution of $\frac{CPE_0^2}{CPE_I^2}$ will therefore be Chi-square, with n-1 degrees of freedom for a random sample of size n fired under identical test conditions.

Identical conditions from round to round are not attainable during testing, however. Therefore, $\frac{CPE_0^2}{CPE_I^2}$ will not have a Chi-square distribution during tests. In fact, CPE_I cannot be adequately estimated from test results, since the effects of round-to-round variability in test conditions will always be included in circular probable error estimates.

To develop acceptance criteria for APDS rounds, it is necessary to know the form of the distribution of CPE_0. It is also necessary to estimate a circular probable error, characterizing a lot, which can be determined from test data.

During acceptance testing, the effect of variation in test conditions from round to round on circular probable error estimates will vary from one occasion to another. On some days, test condition variability has little effect on dispersion patterns, while on other days, the effect of variable test conditions is comparatively large. Obviously, measurements which characterize a lot or its expected dispersion should not be based on days when test condition variability is unusually small or large. The measurement which adequately characterizes the performance of a lot should be based on the outcome expected on a randomly selected day, given that the day satisfies the meteorological requirement for conducting an acceptance test.

To define characteristic circular probable error, i.e., the probable error which is expected from a lot on a random day, we assume that a lot of infinite size is available. Let N random samples be selected from the lot and let each sample be tested on a random day. Let CPE_C denote the characteristic circular probable error of the lot. Then,

$$CPE_C = \lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}(CPE_0^2)_i\right]^{1/2}$$

defines the characteristic circular probable error of the lot. In the above equation, $(CPE_0^2)_i$ is the square of the circular probable error observed on the i^{th} occasion. Since $\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}(CPE_0^2)_i\right]^{1/2}$ equals the expected value of the observed circular probable error on a randomly selected day, it is clear that CPE_C adequately defines the performance of the lot.

On some days, $E(CPE_0) \neq CPE_C$, since variability in test conditions such as wind and weather, tube wear from round to round, droop, jump, etc., may be unusually large or small. Suppose conditions on a given day are such that $E(CPE_0) = CPE_C$ for the random samples fired on that day. Call this an average day.

Now, looking only at tests conducted on average days (days for which $E(CPE_0) = CPE_C$), there is a characteristic radial standard deviation, σ_C, which describes the dispersion characteristics of the lot. Since circular probable error is a multiple of radial standard deviation, then,

$$\frac{CPE_0^2}{CPE_C^2} = \frac{S^2}{\sigma_C^2},$$

where CPE_0 is the circular probable error observed for a random sample tested on an average day, and S^2 is the estimate of the variance. The distribution of $\frac{CPE_0^2}{CPE_C^2}$ will therefore be Chi-square with n-1 degrees of freedom for a random sample of size n tested on an average day.

Acceptance tests, however, are not usually conducted on average days, and

$$\frac{CPE_0^2}{CPE_C^2}$$

has in actuality greater variability than that predicted by the Chi-square distribution. To assess how this ratio varies during acceptance tests, the dispersion results of 176 groups of 105mm, APDS, M392 reference rounds were analyzed. Eight of these groups had target misses and were excluded from further analysis. The remaining groups were tested on 168 different days over a twelve year period. Each group

consisted of a ten round sample from one of two reference lots, and was fired at a target 2000 meters from the gun. There was no significant difference in the distributions of circular probable errors of each reference lot and results from the two lots were therefore combined. The estimate of the characteristic circular probable error of the reference lots, CPE_C, was obtained from the following equation.

$$\hat{CPE}_C = \left[\frac{1}{168} \sum_{i=1}^{168} (CPE_0^2)_i \right]^{1/2},$$

where $(CPE_0^2)_i$ was the square of the circular probable error observed on the i^{th} occasion.

In order to facilitate the analysis, $\frac{CPE_0}{\hat{CPE}_C}$ was defined to be λ. Assuming that CPE_C for the 168 reference groups was equal to the characteristic circular probable error of the reference lot, then

$$\lambda_i = \frac{CPE_{0\ i}}{\hat{CPE}_C}.$$

For each of the 168 groups, λ_i was determined and the observed cumulative distribution of the λ_i's was plotted. Figure 2 presents the observed cumulative distribution of λ_i and compares it to the cumulative distribution which would result if λ_i^2 were distributed as a Chi-square with nine degrees of freedom.

From Figure 2, it is evident that if λ_i^2 were distributed as a Chi-square distribution with nine degrees of freedom, 80 percent of the observations would be between 0.68 and 1.28. The observed cumulative distribution of the λ_i's, however, shows that 80 percent of the observations were between 0.46 and 1.44, a considerably wider spread than that predicted by the Chi-square.

The Gamma distribution of the form Gamma $(X) = \frac{X^{\alpha-1}}{(\alpha-1)!\beta^\alpha} e^{-\frac{X}{\beta}}$ was fitted to the observed λ_i's. With $\alpha = 7.4558$ and $\beta = 0.1261$, the Gamma distribution fits the data very well. The Chi-square, Cramer-Von Mises and Kolmogorov-Smirnov goodness of fit tests gave no reason to reject the Gamma distribution. A summary of the observed λ_i's and of the fitted Gamma distribution are presented in Figure 3. Each data

point, denoted by a triangle, represents the number of observations within an interval of length 0.10. Points from the Gamma distribution were multiplied by 16.8 so that the data and fitted curve could be shown on the same scale. In Figure 4, the cumulative Gamma distribution is compared with the observed cumulative distribution of the λ_i's. The data points, denoted by triangles, represent the observed cumulative probability that $\lambda_i \leq \lambda$.

4.3 Criteria For Acceptance Plans

Assuming that λ follows the Gamma distribution with $\alpha = 7.4558$ and $\beta = 0.1261$ for random samples of size 10, acceptance plans can be derived with various levels of consumer's risks (probability of accepting a lot with poor quality) and producer's risks (probability of rejecting a lot with good quality). The limitation of using this Gamma distribution is that it can only be used to calculate consumer's and producer's risks for acceptance plans with sample sizes which are multiples of ten. Derivations of distributions applicable to sample sizes other than multiples of ten are beyond the scope of this report. To develop an acceptance plan, it is necessary to specify the levels of characteristic circular probable errors associated with both good and poor quality. The sampling procedures and associated decision criteria must then be designated. Once this is done, the Gamma distribution can be used to derive the operating characteristics (OC), i.e., the probability of accepting a lot with a specified quality, associated with the plan. Comparisons of the OC of various plans can also be made, and the best plan can thereby be determined.

4.4 Development of Acceptance Plans

To develop the acceptance plans, CPE_g is defined as the level of circular probable error which represents good quality of a lot. Poor lot quality can be characterized by any multiple of CPE_g, as long as the multiple is greater than one. In order to develop examples of acceptance plans it is assumed in this report that poor quality is characterized by values of circular probable error greater than 2 CPE_g. One possible acceptance plan is to test a ten round random sample from a lot and to calculate the observed circular probable error, CPE_0, and then, decide to accept or reject the lot. One set of decision criteria includes accepting the lot, if $CPE_0 < 1.2\ CPE_g$ or rejecting the lot, if $CPE_0 \geq 1.2\ CPE_g$. This plan will be designated as Acceptance Plan A. Given a circular probable error, CPE_L, which characterizes the lot, $\frac{CPE_0}{CPE_L}$ is distributed as the Gamma distribution discussed in section 4.2, provided CPE_L is not very different from the pooled circular probable

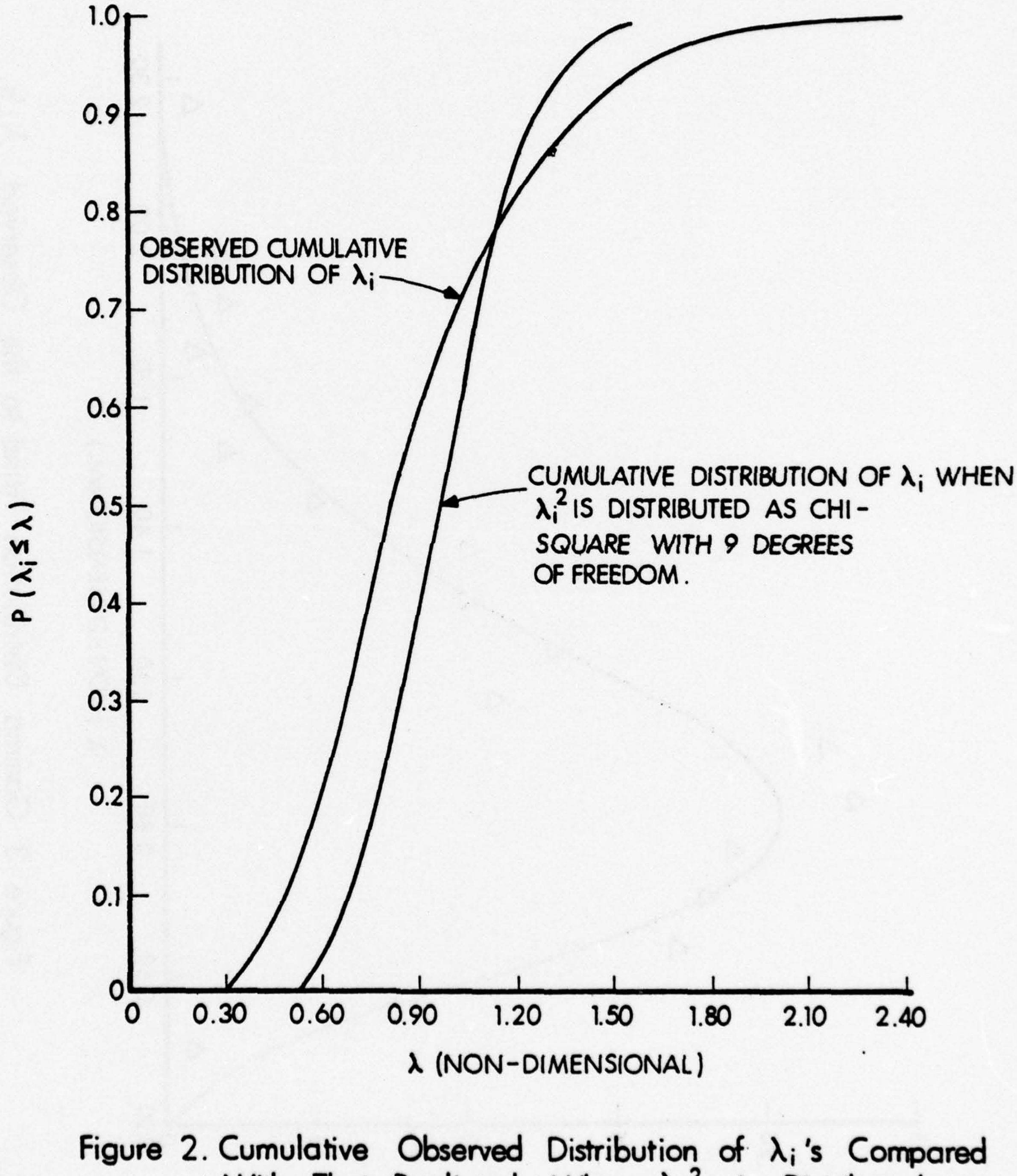

Figure 2. Cumulative Observed Distribution of λ_i's Compared With That Predicted When λ_i^2 is Distributed as Chi-Square With 9 Degrees of Freedom.

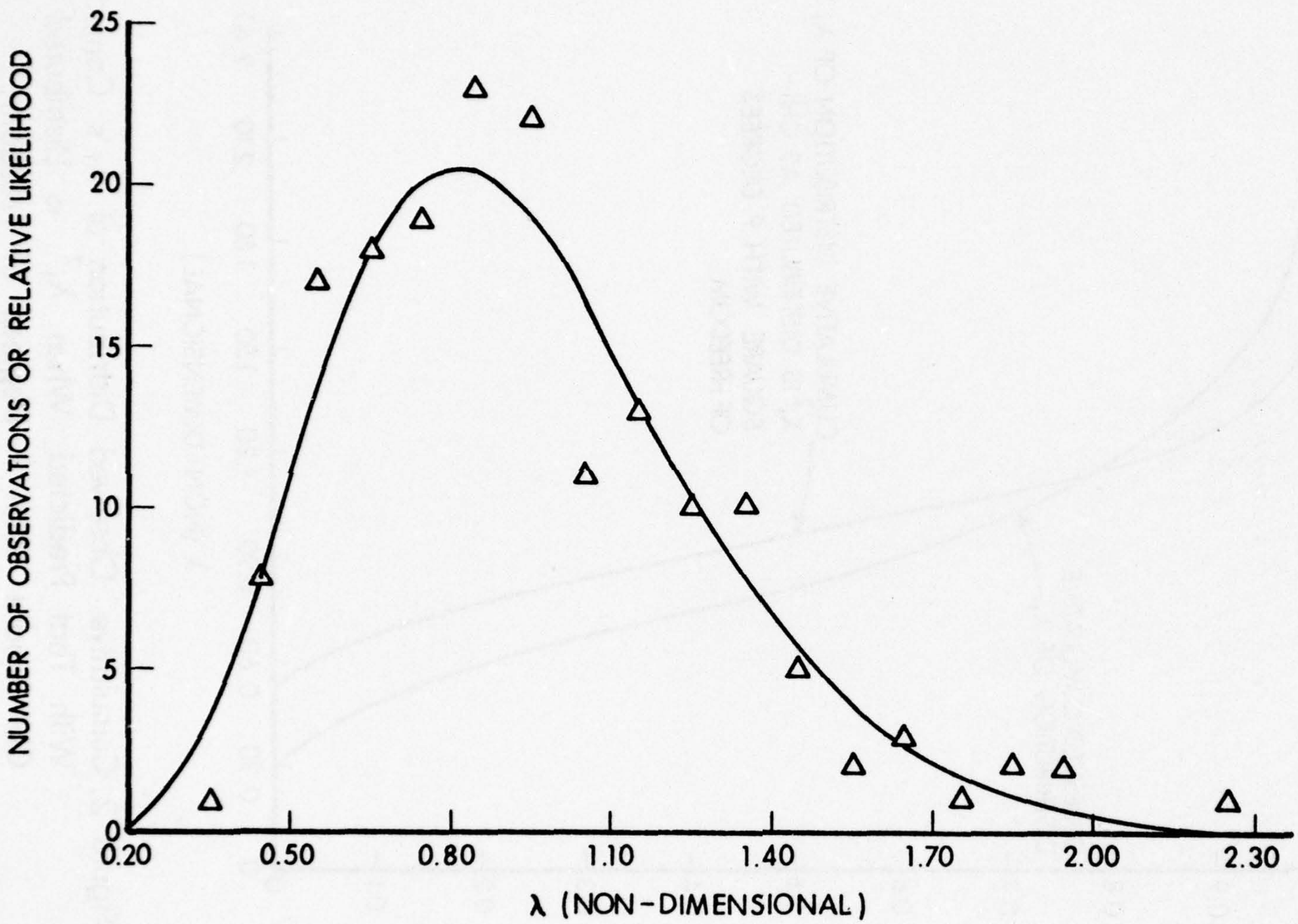

Figure 3. Gamma Distribution Fitted to the Observed λ_i's.

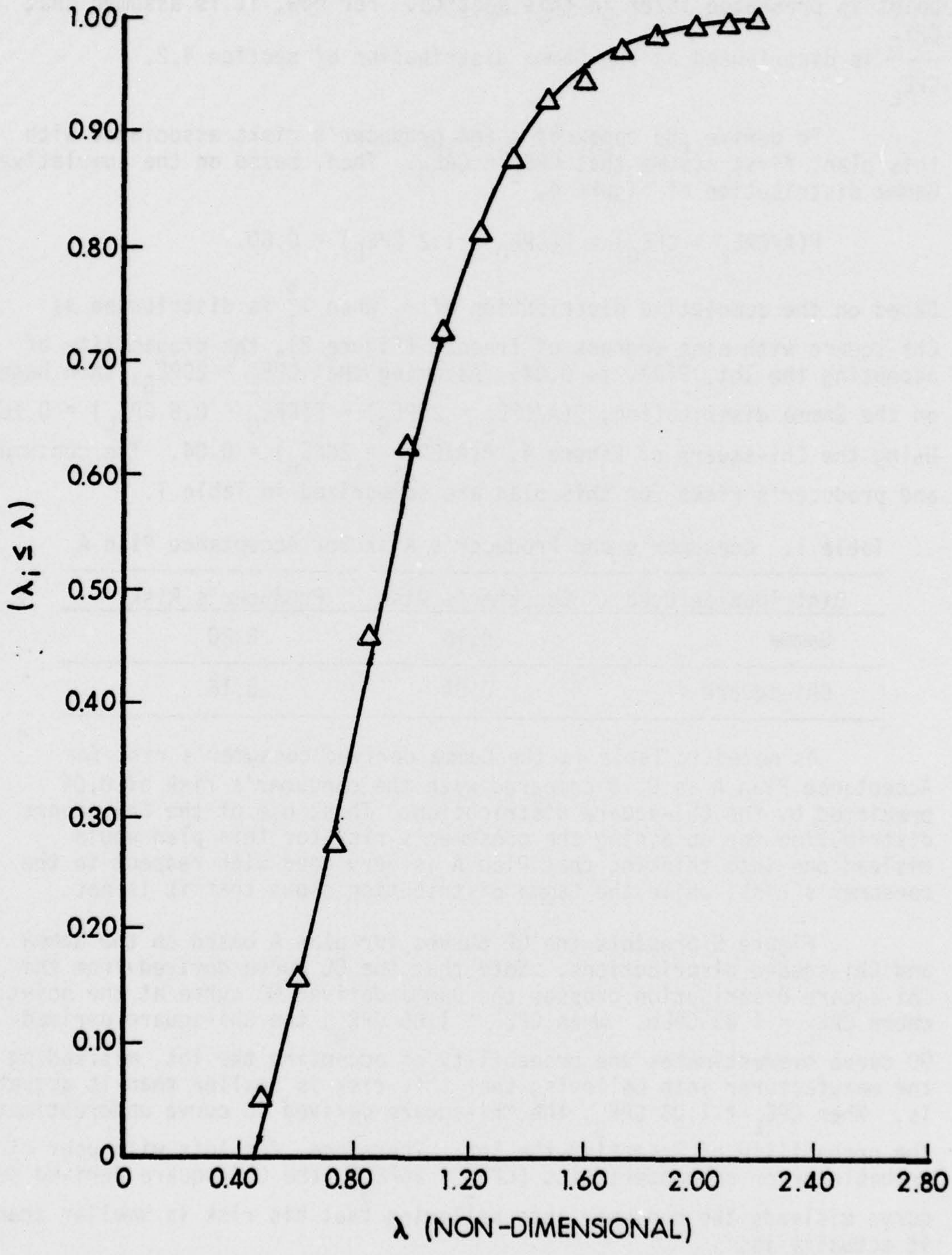

Figure 4. Cumulative Gamma Distribution Fitted to the Observed λ_i's.

error of the 168 reference round groups. Further discussion of this point is presented later in this section. For now, it is assumed that $\frac{CPE_0}{CPE_L}$ is distributed as the Gamma distribution of section 4.2.

To derive the consumer's and producer's risks associated with this plan, first assume that $CPE_L = CPE_G$. Then, based on the cumulative Gamma distribution of Figure 4,

$$P(A/CPE_L = CPE_G) = P(CPE_0 < 1.2\ CPE_L) = 0.80.$$

Based on the cumulative distribution of λ_i when λ_i^2 is distributed as Chi-square with nine degrees of freedom (Figure 2), the probability of accepting the lot, P(A), is 0.84. Assuming that $CPE_L = 2CPE_G$, then based on the Gamma distribution, $P(A/CPE_L = 2CPE_G) = P(CPE_0 < 0.6\ CPE_L) = 0.16$. Using the Chi-square of Figure 4, $P(A/CPE_L = 2CPE_G) = 0.04$. The consumer's and producer's risks for this plan are summarized in Table 1.

Table 1. Consumer's and Producer's Risk For Acceptance Plan A

Distribution Used	Consumer's Risk	Producer's Risk
Gamma	0.16	0.20
Chi-square	0.04	0.16

As noted in Table 1, the Gamma derived consumer's risk for Acceptance Plan A is 0.16 compared with the consumer's risk of 0.04 predicted by the Chi-square distribution. Thus, use of the Chi-square distribution for obtaining the consumer's risk for this plan would mislead one into thinking that Plan A is very good with respect to the consumer's risk, while the Gamma distribution shows that it is not.

Figure 5 presents the OC curves for plan A based on the Gamma and Chi-square distributions. Note that the OC curve derived from the Chi-square distribution crosses the Gamma derived OC curve at the point where $CPE_L = 1.06$ CPEg. When $CPE_L < 1.06\ CPE_g$, the Chi-square derived OC curve overestimates the probability of accepting the lot, misleading the manufacturer into believing that this risk is smaller than it actually is. When $CPE_L > 1.06\ CPE_g$, the Chi-square derived OC curve underestimates the probability of accepting the lot. Therefore, for lots with poor circular probable error characteristics ($CPE_L > 2CPE_g$), the Chi-square derived OC curve misleads the consumer into believing that his risk is smaller than it actually is.

In Table 2 several acceptance plans are presented. It should be noted that these plans represent only a finite subset of an infinite set of acceptance plan strategies.

Table 2. Acceptance Plan Alternatives B Through H Based On Multiples Of Ten Round Samples

Acceptance Plan Designation	Description of Acceptance Plan
B	Test a single ten round random sample from a lot. Accept the lot if $CPE_o < 1.4$ CPEg. Otherwise reject the lot.
C	Test a single ten round random sample from a lot. Accept the lot if $CPE_o < 1.6$ CPEg. Otherwise reject the lot.
D	Test a single ten round random sample from a lot. Accept the lot if $CPE_o < 1.8$ CPEg. Otherwise reject the lot.
E	Test two ten round random samples from a lot. If the pooled $CPE_o < 1.4\ CPE_g$, accept the lot. Otherwise reject the lot.
F	Test a ten round random sample from a lot. Accept the lot if $CPE_o < 1.4\ CPE_g$. Reject the lot if $CPE_o > 1.8\ CPE_g$. Otherwise, test a second ten round random sample. Then, accept the lot if the pooled $CPE_o < 1.4\ CPE_g$. If the pooled $CPE_o \geq 1.4\ CPE_g$, reject the lot.
G	Test a ten round random sample from a lot. Accept the lot if $CPE_o < 1.2\ CPE_g$. Reject the lot if $CPE_o > 1.54\ CPE_g$. Otherwise, test a second ten round random sample. Then, accept the lot if the pooled $CPE_o < 1.2\ CPE_g$. If the pooled $CPE_o \geq 1.2\ CPE_g$, reject the lot.
H	Test a ten round random sample from a lot. Accept the lot if $CPE_o < 1.6\ CPE_g$. Reject the lot if $CPE_o > 2.0\ CPE_g$. Otherwise, test a second ten round random sample. Then, accept the lot if the pooled $CPE_o < 1.6\ CPE_g$. If the pooled $CPE_o \geq 1.6\ CPE_g$, reject the lot.

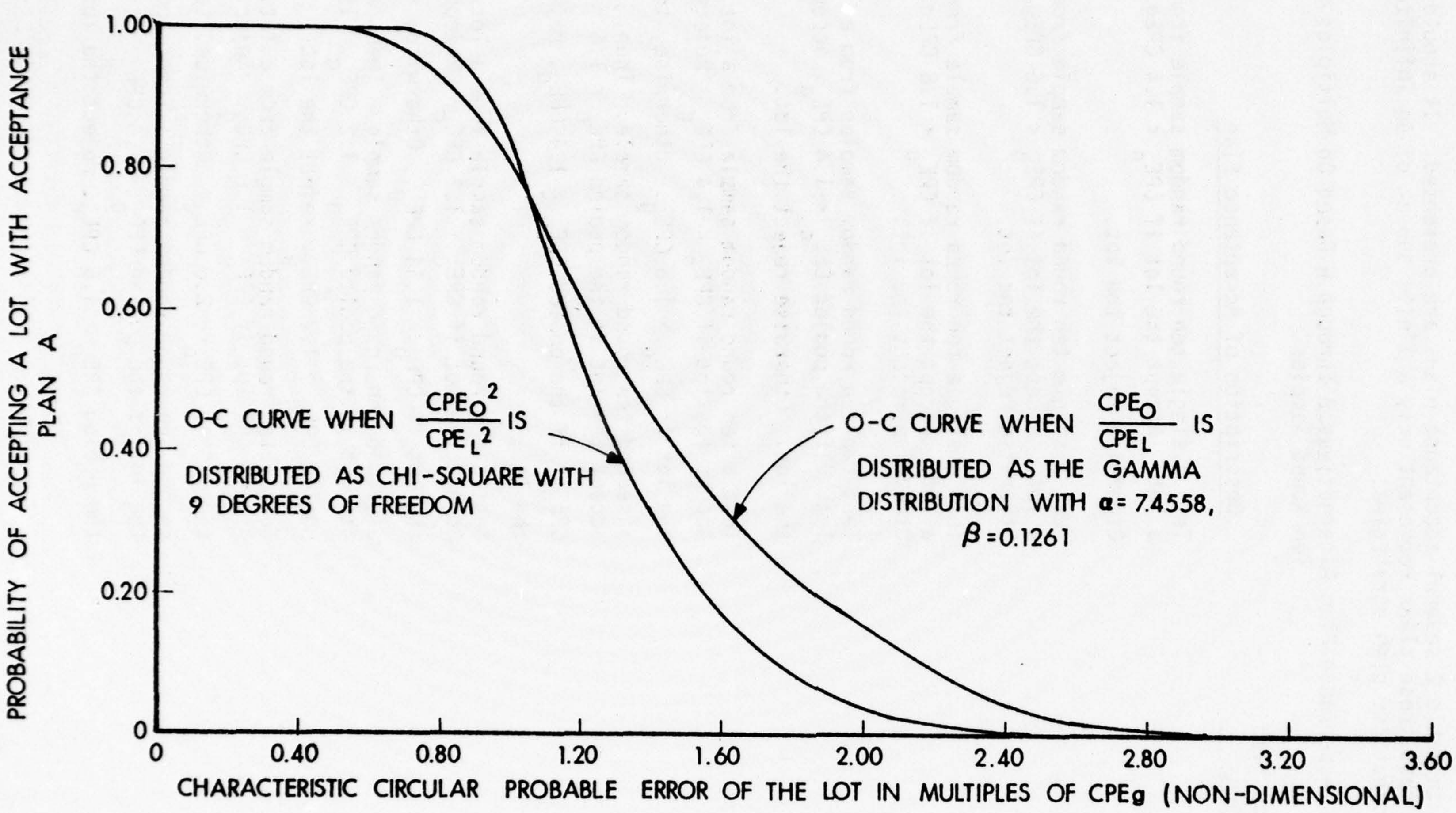

Figure 5. Operating Characteristics of Acceptance Plan A.

Figures 6 through 12 present the OC curves for acceptance plans B through H described in Table 2. In each figure, the consumer's risk is determined from the probability of accepting the lot when $CPE_L = 2\ CPE_g$. When $CPE_L = CPE_g$, the producer's risk is given by 1-P(A), where P(A) is the probability of accepting the lot. The OC curves for all of the plans show that the Chi-square derived consumer's and producer's risks are lower than those derived from the Gamma distribution.

Table 3 summarizes the consumer's and producer's risks for acceptance plans A through H. The consumer's risks are presented as ranges. The lower bound in each case is obtained from the Chi-square derived OC curve, while the upper bound is obtained from the Gamma derived OC curve. The producer's risks are presented as point estimates and are obtained from the Gamma derived OC curves.

Table 3. Consumer's and Producer's Risks for Acceptance Plans A Through H

Acceptance Plan Designation	Consumer's Risk	Producer's Risk
A	0.04 - 0.16	0.20
B	0.11 - 0.25	0.10
C	0.24 - 0.40	0.13
D	0.38 - 0.50	0.07
E	0.04 - 0.09	0.07
F	0.12 - 0.28	0.04
G	0.04 - 0.16	0.11
H	0.27 - 0.55	0.02

It was previously assumed that $\frac{CPE_0}{CPE_L}$ would be distributed as the Gamma distribution with $\alpha = 7.4558$ and $\beta = 0.1261$. If CPE_L equals the characteristic circular probable error of the reference lot used to obtain the fitted Gamma distribution, the assumption is reasonable. The Gamma distribution was obtained from the actual distribution of observed circular probable errors over a twelve year period. It is reasonable to assume that this is representative of the distribution which will occur in the future. However, as CPE_L deviates from the characteristic circular probable error of the reference lots, the distribution of CPE_0 deviates from the Gamma distribution. The derived Gamma distribution represents the deviation in observed circular probable errors due to both inherent round-to-round differences and occasion-to-occasion test condition differences. As inherent round-to-round differences increase, which is the case for poor quality control during manufacturing, they tend to have an increasingly dominating effect on the distribution of CPE_0 relative

to the effect of occasion-to-occasion test condition variation. As the inherent differences increase without bound, the relative effect of occasion-to-occasion variation in test conditions upon CPE_O tends towards zero, and the distribution of $\frac{CPE_O^2}{CPE_L^2}$ approaches the Chi-square. On the other hand, as inherent round-to-round differences decrease, the scatter of observed probable errors is increasingly dominated by the effect of test condition variation, and $\frac{CPE_O}{CPE_L}$ will tend to have greater variation than that predicted by the Gamma distribution. Consequently, as CPE_L increases towards poor lot quality, the true probability of accepting the lot lies somewhere between the probabilities obtained from the Chi-square and Gamma derived OC curves as shown in Figures 5 through 12. As CPE_L improves beyond good lot quality, the probability of accepting the lot decreases below that predicted by the Gamma derived OC curve. For these reasons, the consumer's risks in Table 3 are presented as ranges, with the Chi-square and Gamma derived risks being the lower and upper bounds, respectively. The producer's risk, since it is based upon lots of good quality, is presented as a point estimate based upon the Gamma derived OC curve. If CPE_g is approximately equal to the characteristic circular probable error of the reference rounds, the Gamma derived risk is a reasonably good estimate of the producer's risk. However, if CPE_g is better than the characteristic circular probable error of the reference rounds, the Gamma derived producer's risk is a lower bound. This could happen if future APDS rounds are markedly more accurate than the reference rounds.

Reviewing the acceptance plans in Table 3, it is evident that Plan E provides the best combination of consumer's and producer's risks. However, twenty rounds are always needed for this plan. Plan G is the next best plan and is less costly than Plan E. If lots are produced with CPE_L equal to CPE_g, the average sample size for this plan is 11.5. This is due to the fact that retests would occur only 15 percent of the time.

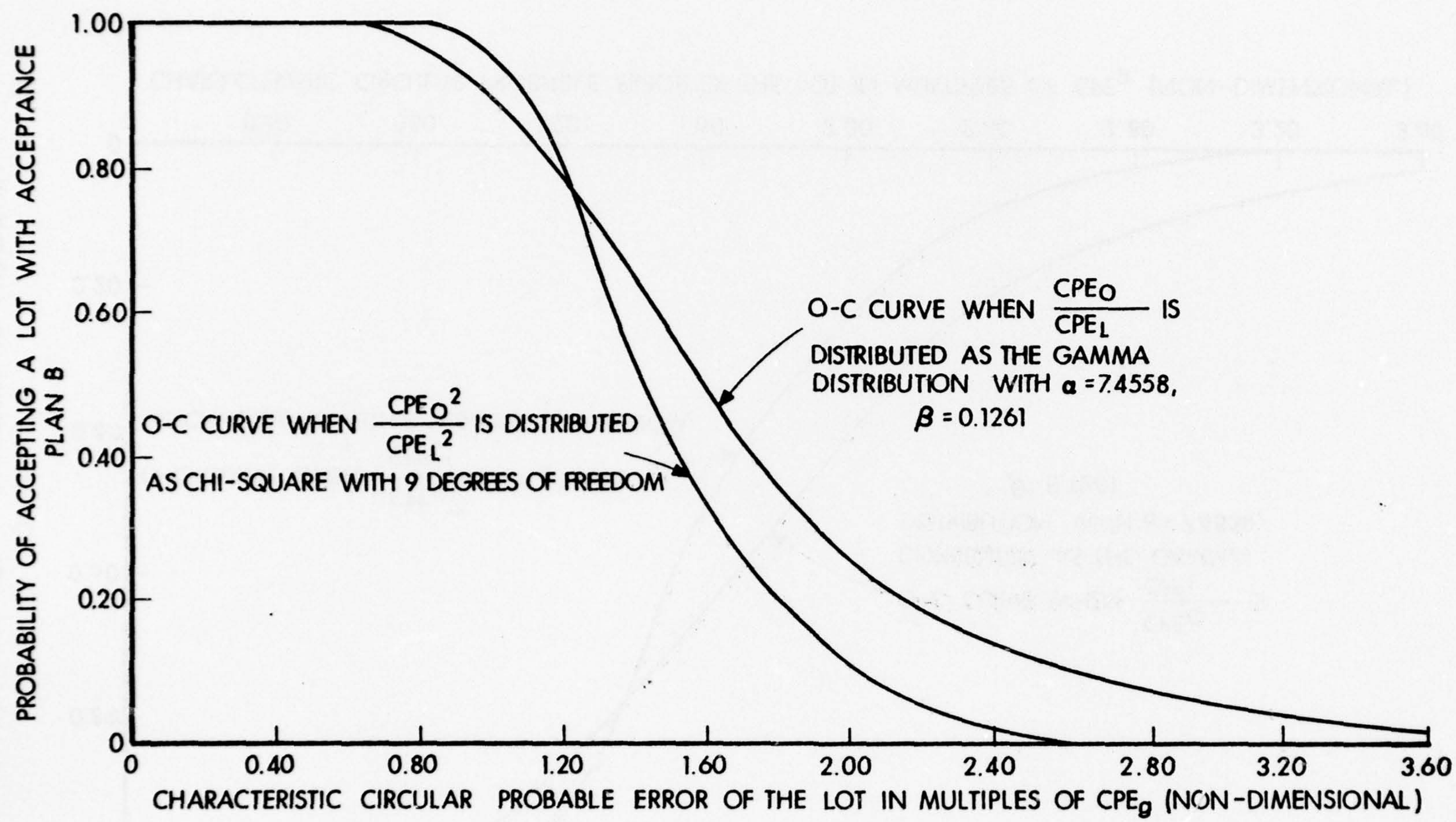

Figure 6. Operating Characteristic of Acceptance Plan B.

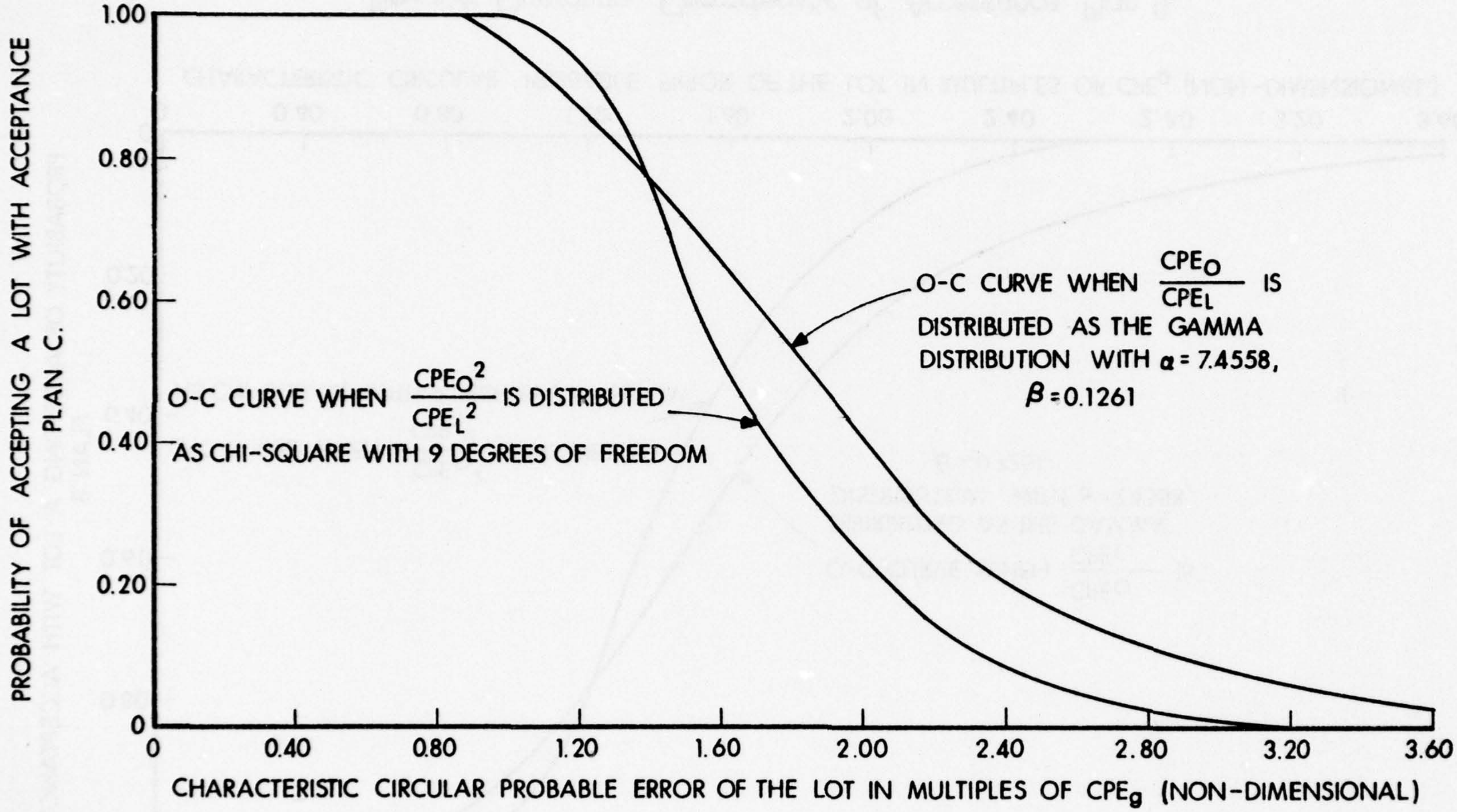

Figure 7. Operating Characteristic of Acceptance Plan C.

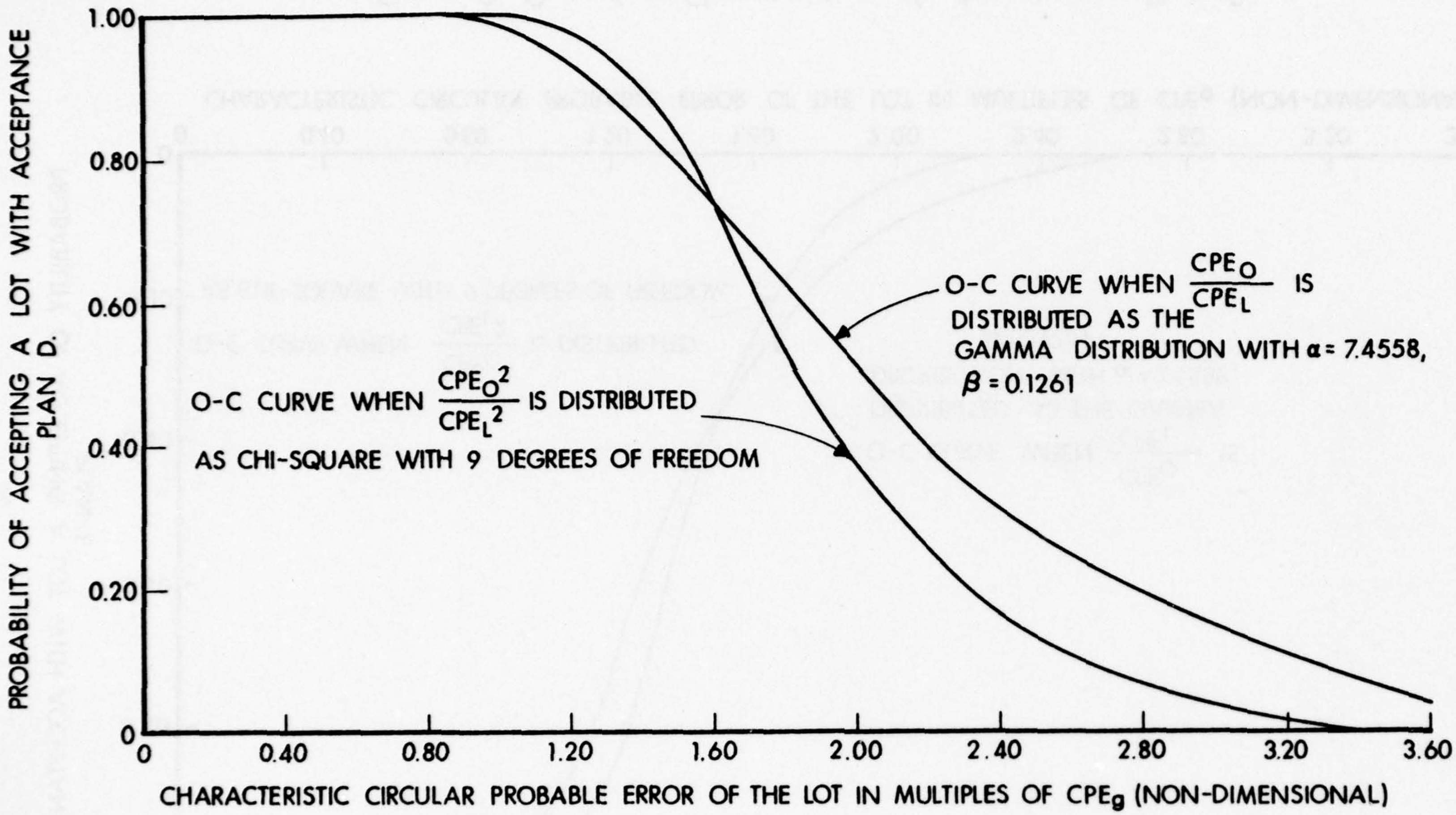

Figure 8. Operating Characteristic of Acceptance Plan D.

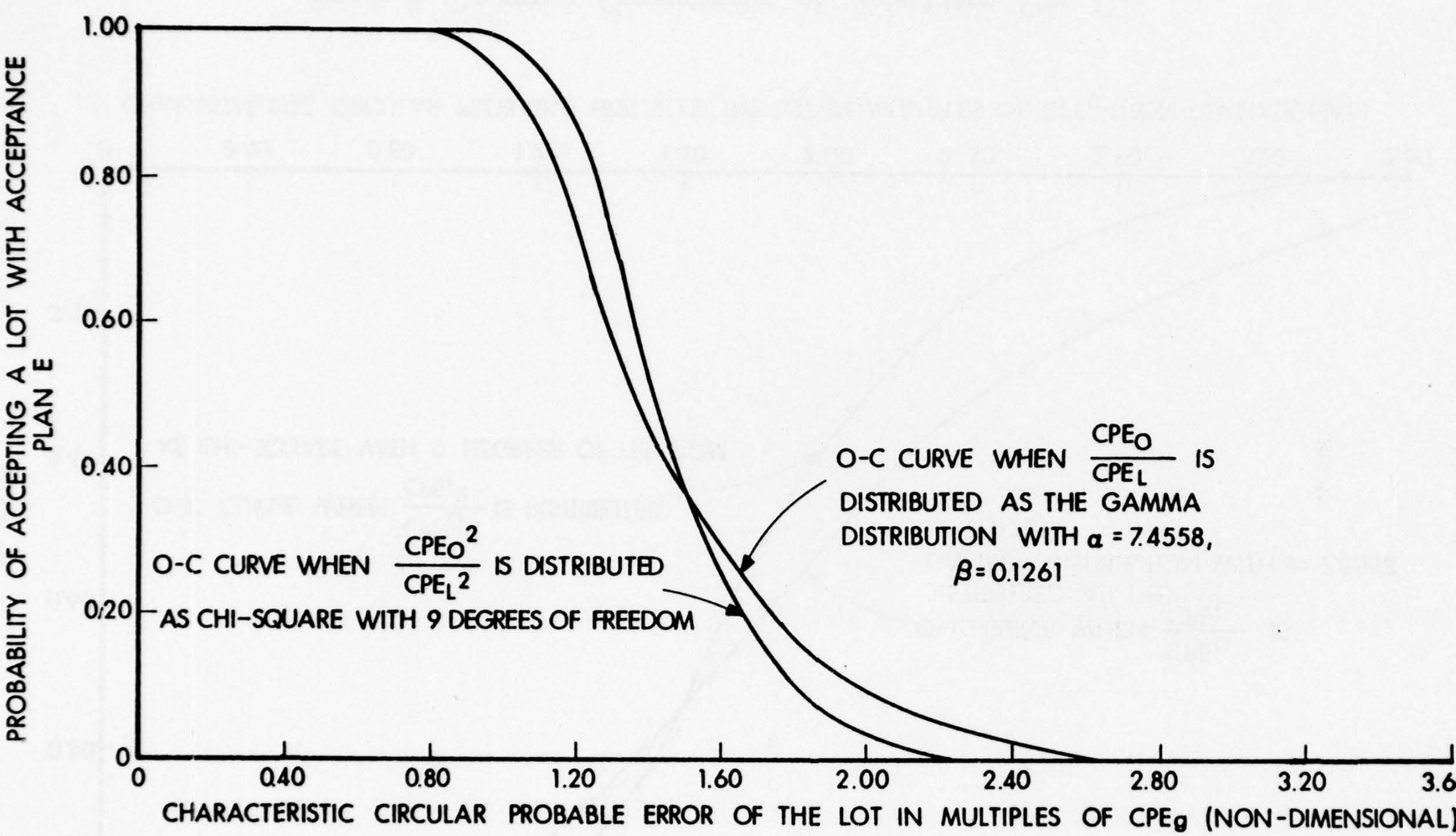

Figure 9. Operating Characteristics of Acceptance Plan E.

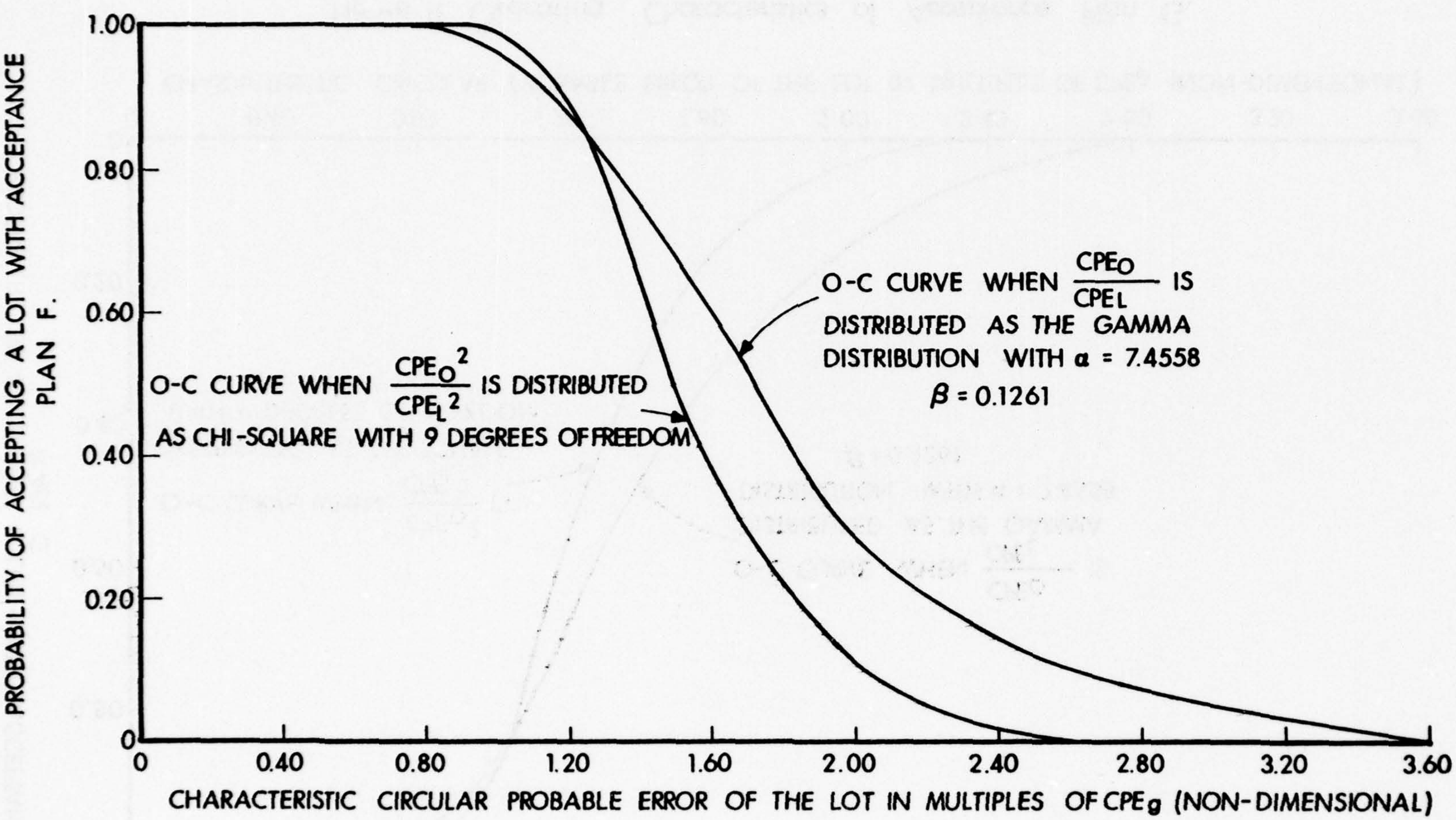

Figure 10. Operating Characteristics of Acceptance Plan F.

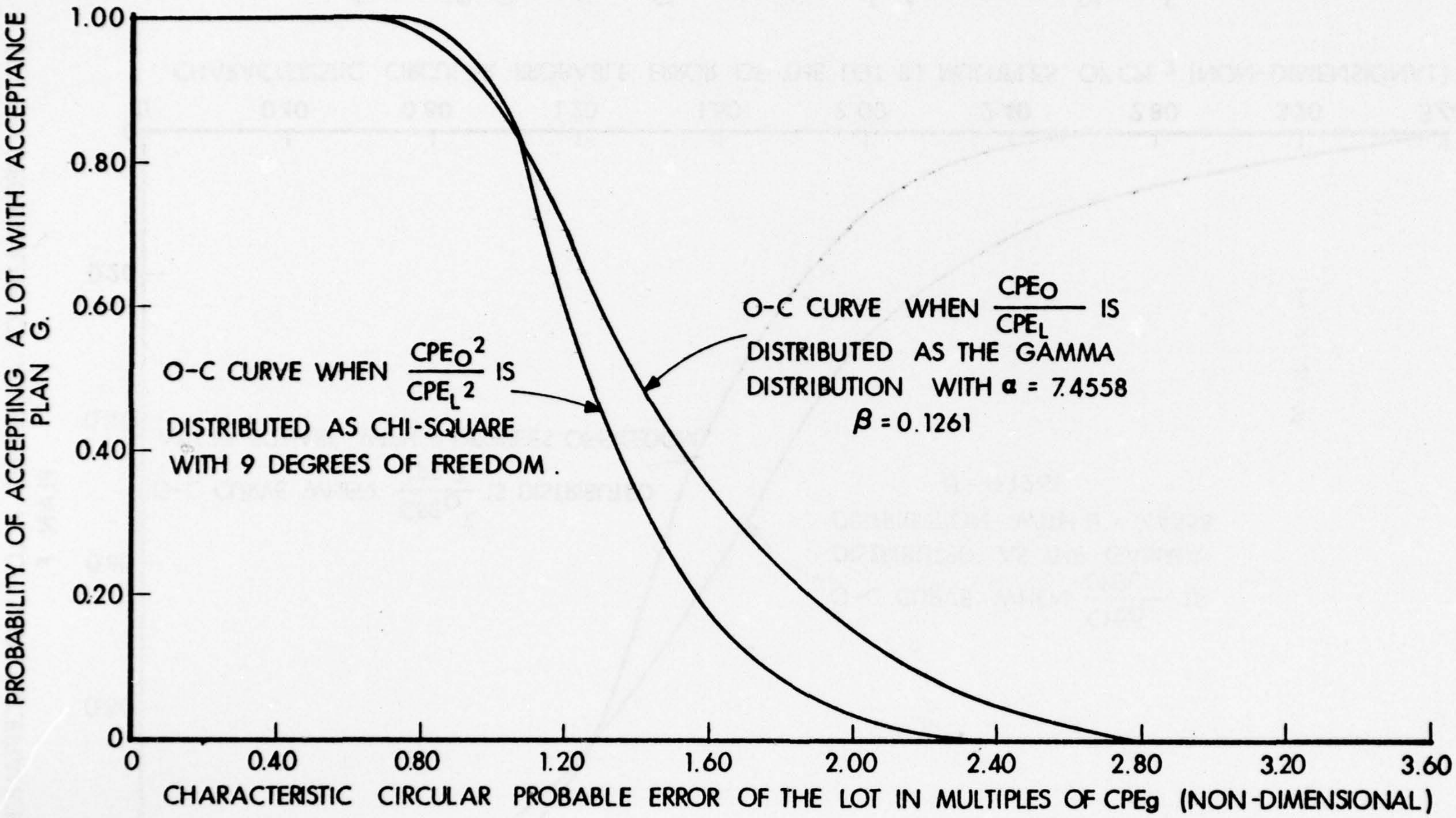

Figure 11. Operating Characteristics of Acceptance Plan G.

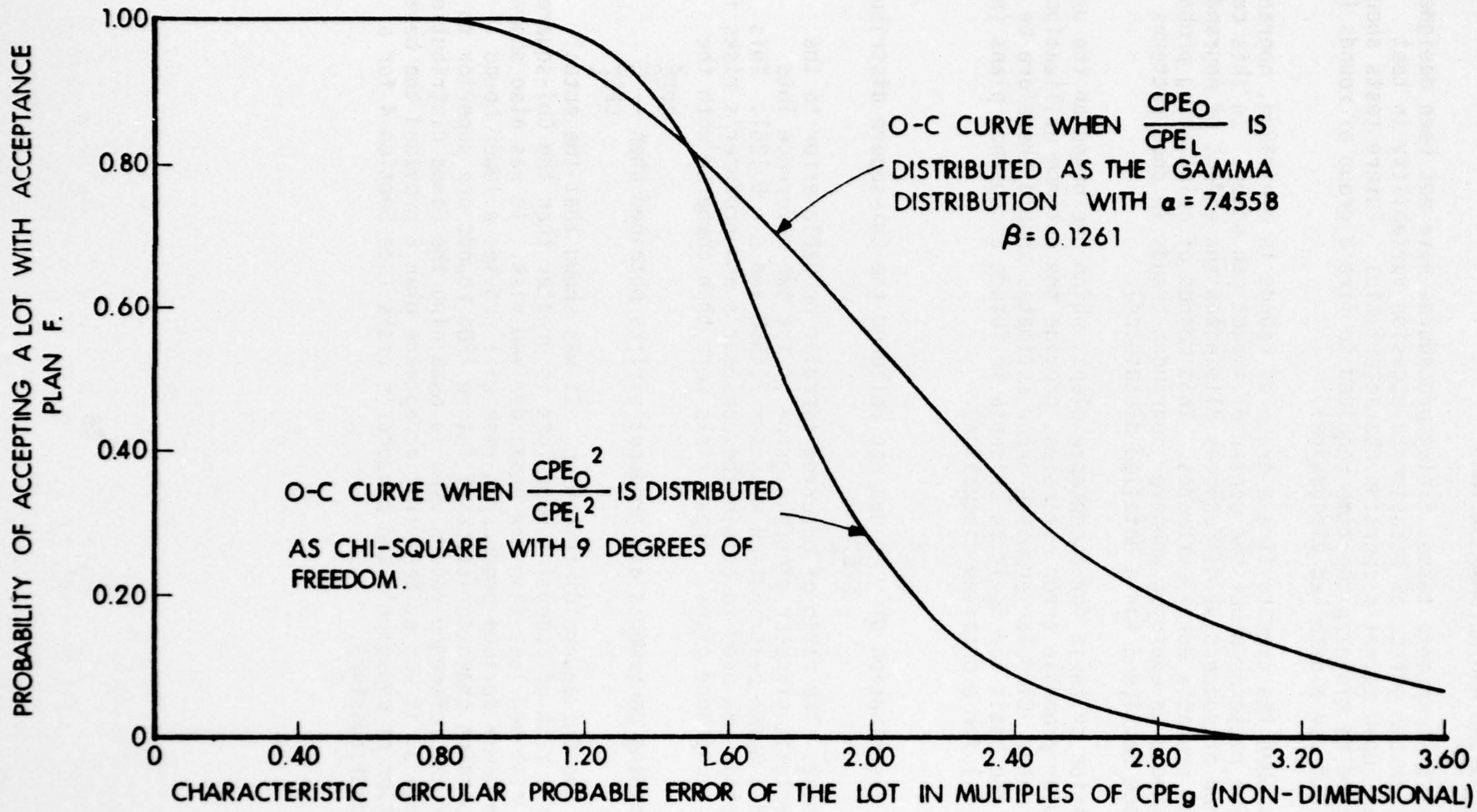

Figure 12. Operating Characteristics of Acceptance Plan H.

5. CONCLUSIONS AND RECOMMENDATIONS

In past acceptance tests, firing procedures have not been designed to minimize the effect of occasion-to-occasion variability in test conditions upon target dispersion characteristics. Future tests should be designed to minimize the time required to fire a group of rounds (see Section 2.2 for a detailed discussion).

Even when the time to fire a group of rounds is minimized, nonrandom dispersion patterns about the center of impact can occur. In this case the method of successive differences eliminates the effect of nonrandom trends on probable error estimates. This method of calculating probable errors should be employed whenever nonrandom trends in shot patterns occur (see Section 3 for a detailed discussion).

Decision criteria for acceptance plans, which are based on the use of circular probable error estimates, provide the optimum utilization of test data. Circular probable error estimates should therefore be used as the basis for decision criteria in future acceptance plans (see Section 4.1 for a detailed discussion).

The distribution of $\dfrac{CPE_o^2}{CPE_L^2}$ has not followed the Chi-square distribution in the past. The ratio of observed circular probable error to the characteristic circular probable error of the two reference lots followed a Gamma distribution with $\alpha = 7.4558$ and $\beta = 0.1261$. This distribution was used to obtain the consumer's and producer's risks for several acceptance plans. These risks were then compared with the corresponding consumer's and producer's risks obtained when $\dfrac{CPE_o^2}{CPE_L^2}$ follows the Chi-square distribution. It was shown that the actual consumer's risk of accepting poor lots is greater than the Chi-square derived risk and less than the Gamma derived risk. It was also shown that the Gamma derived producer's risk will provide a lower bound if the dispersion characteristics of future APDS rounds are superior to those of the reference rounds used in obtaining the Gamma distribution. From Table 3, it was evident that acceptance plan E provided the best combination of consumer's and producer's risks (see Section 4 for a detailed discussion).

The acceptance plans presented in this report represent only a small portion of the plans which could be developed for APDS type of ammunition. Undoubtedly it will be necessary to develop different acceptance plans for new types of APDS rounds undergoing development. These plans should be developed with recognition that the OC curves underestimate both the consumer's and producer's risks, assuming that $\frac{CPE_0^2}{CPE_L^2}$ follows the Chi-square distribution. The gamma distribution should be used for developing OC curves for the various plans. The consumer's and producer's risks associated with each plan can be compared and the best plan can then be selected (see Section 4.3 for a detailed discussion).

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APPENDIX

Horizontal, vertical and circular probable errors observed for two reference lots of 105mm, APDS, M392 rounds on 168 occasions. A sample size of ten was used on each occasion.

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	Probable Error In Mils		
Occasion	Horizontal	Vertical	Circular
1	0.47	0.20	0.62
2	0.14	0.29	0.39
3	0.25	0.24	0.42
4	0.28	0.23	0.44
5	0.28	0.18	0.41
6	0.12	0.13	0.22
7	0.19	0.15	0.30
8	0.16	0.13	0.25
9	0.35	0.25	0.53
10	0.21	0.15	0.32
11	0.10	0.18	0.25
12	0.21	0.13	0.30
13	0.13	0.06	0.18
14	0.26	0.12	0.35
15	0.20	0.16	0.31
16	0.14	0.24	0.34
17	0.20	0.15	0.31
18	0.30	0.17	0.42
19	0.23	0.16	0.34
20	0.09	0.16	0.22
21	0.25	0.15	0.36
22	0.19	0.10	0.26
23	0.30	0.14	0.40
24	0.05	0.12	0.16
25	0.14	0.11	0.22
26	0.14	0.23	0.33
27	0.22	0.28	0.44
28	0.10	0.12	0.19
29	0.08	0.16	0.22
30	0.14	0.17	0.27
31	0.16	0.13	0.25
32	0.23	0.33	0.49
33	0.11	0.08	0.17
34	0.16	0.21	0.32
35	0.32	0.18	0.45
36	0.13	0.14	0.23
37	0.10	0.18	0.25
38	0.18	0.16	0.29
39	0.26	0.29	0.48
40	0.24	0.15	0.35
41	0.37	0.15	0.49
42	0.47	0.20	0.62

Occasion	Probable Error In Mils Horizontal	Vertical	Circular
43	0.24	0.33	0.50
44	0.26	0.33	0.51
45	0.22	0.18	0.35
46	0.14	0.29	0.39
47	0.39	0.17	0.52
48	0.26	0.16	0.37
49	0.18	0.20	0.33
50	0.10	0.23	0.31
51	0.20	0.20	0.35
52	0.18	0.22	0.35
53	0.18	0.15	0.29
54	0.12	0.08	0.18
55	0.30	0.14	0.40
56	0.22	0.20	0.36
57	0.25	0.25	0.43
58	0.12	0.13	0.22
59	0.17	0.18	0.30
60	0.20	0.14	0.30
61	0.15	0.15	0.26
62	0.33	0.30	0.55
63	0.37	0.30	0.58
64	0.29	0.29	0.50
65	0.38	0.23	0.54
66	0.20	0.18	0.33
67	0.20	0.14	0.30
68	0.25	0.15	0.36
69	0.31	0.13	0.41
70	0.16	0.12	0.24
71	0.18	0.33	0.46
72	0.21	0.15	0.32
73	0.22	0.19	0.36
74	0.33	0.27	0.52
75	0.14	0.07	0.19
76	0.15	0.09	0.21
77	0.10	0.27	0.35
78	0.11	0.09	0.17
79	0.10	0.13	0.20
80	0.06	0.09	0.13
81	0.26	0.17	0.38
82	0.16	0.13	0.25
83	0.12	0.11	0.20
84	0.14	0.21	0.31

	Probable Error In Mils		
Occasion	Horizontal	Vertical	Circular
85	0.32	0.14	0.43
86	0.20	0.14	0.30
87	0.19	0.18	0.32
88	0.11	0.08	0.17
89	0.15	0.11	0.23
90	0.30	0.14	0.40
91	0.13	0.18	0.27
92	0.12	0.18	0.26
93	0.22	0.19	0.36
94	0.11	0.12	0.20
95	0.21	0.16	0.32
96	0.09	0.11	0.17
97	0.15	0.22	0.33
98	0.24	0.13	0.33
99	0.30	0.17	0.42
100	0.12	0.11	0.20
101	0.11	0.11	0.19
102	0.26	0.09	0.34
103	0.25	0.15	0.36
104	0.18	0.15	0.29
105	0.26	0.17	0.38
106	0.10	0.19	0.26
107	0.22	0.16	0.33
108	0.20	0.07	0.26
109	0.22	0.32	0.48
110	0.17	0.15	0.28
111	0.20	0.16	0.31
112	0.25	0.19	0.38
113	0.30	0.21	0.45
114	0.14	0.10	0.21
115	0.18	0.09	0.25
116	0.09	0.16	0.22
117	0.15	0.18	0.29
118	0.21	0.20	0.35
119	0.20	0.53	0.69
120	0.16	0.16	0.28
121	0.21	0.18	0.34
122	0.19	0.13	0.28
123	0.11	0.06	0.15
124	0.10	0.13	0.20
125	0.12	0.18	0.26
126	0.18	0.16	0.29

	Probable Error In Mils		
Occasion	Horizontal	Vertical	Circular
127	0.22	0.15	0.33
128	0.13	0.11	0.21
129	0.34	0.17	0.47
130	0.22	0.27	0.43
131	0.29	0.31	0.52
132	0.40	0.17	0.53
133	0.50	0.34	0.74
134	0.43	0.32	0.66
135	0.54	0.44	0.85
136	0.10	0.17	0.24
137	0.33	0.31	0.55
138	0.30	0.27	0.49
139	0.27	0.27	0.47
140	0.38	0.14	0.50
141	0.43	0.42	0.74
142	0.40	0.31	0.62
143	0.17	0.43	0.57
144	0.18	0.14	0.28
145	0.33	0.26	0.51
146	0.25	0.14	0.35
147	0.20	0.28	0.42
148	0.07	0.18	0.24
149	0.18	0.11	0.26
150	0.19	0.13	0.28
151	0.16	0.23	0.34
152	0.29	0.24	0.46
153	0.19	0.24	0.37
154	0.35	0.32	0.58
155	0.18	0.14	0.28
156	0.15	0.22	0.33
157	0.24	0.08	0.31
158	0.25	0.22	0.41
159	0.25	0.15	0.36
160	0.31	0.23	0.47
161	0.21	0.14	0.31
162	0.26	0.23	0.42
163	0.23	0.37	0.53
164	0.29	0.21	0.44
165	0.16	0.23	0.34
166	0.35	0.44	0.69
167	0.28	0.18	0.41
168	0.17	0.25	0.37
Average	0.22	0.19	0.38

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